DEVELOPMENT OF ELEMENTARY ALGEBRAIC REASONING BY MODELLING PROPORTIONALITY TABLES WITH EXCEL SPREADSHEETS

Rosa Cecilia GAITA IPARRAGUIRRE
Research Institute on Mathematics Education - Pontificia Universidad Católica del Perú
Lima, Lima, CP 1761, Peru

Segundo Ramón MENDOZA ANCAJIMA
Research Institute on Mathematics Education - Pontificia Universidad Católica del Perú
Lima, Lima, CP 1761, Peru

Francisco Javier UGARTE GUERRA
Research Institute on Mathematics Education - Pontificia Universidad Católica del Perú
Lima, Lima, CP 1761, Peru

ABSTRACT

Elementary Algebraic Reasoning (EAR) can be developed from the early grades of schooling, up to consolidated levels in higher education. The notion of proportionality allows the evolution of EAR at different educational levels, starting with the study of tables of values with pencil and paper, to the construction of the notion of linear function and then of linear transformation. This work focuses on the design, experimentation and analysis of responses to didactic situations on proportionality, modelled by means of dynamic tables with the use of Excel spreadsheets. Through the use of dragging, a temporary tool is incorporated between the numerical language, a characteristic of static tables, and the symbolic language, an essential characteristic of a consolidated level of algebraization. The results obtained with 11-12-year-old students show the potential of spreadsheets to develop EAR, allowing the modification of arguments based initially on local generalizations to those involving global generalizations. In this process, there is also an evolution in the language used, from verbal to symbolic.

Keywords: Proportionality, tables of values, elementary algebraic reasoning, spreadsheet.

INTRODUCTION

Elementary algebra is the foundation on which the modern mathematical edifice is built. However, the algebraic curriculum shaped by the process of didactic transposition for more than a century is no longer capable of this essential function (Strømskag & Chevallard, 2022). On the other hand, the generation of situations that allow the evolution of Elementary Algebraic Reasoning (EAR) to evolve throughout schooling is a topic of interest for the community of mathematics educators (Godino et al., 2012; Aké & Godino, 2018; Burgos & Godino, 2019; Burgos & Godino, 2021).

Blanton & Kaput (2003; 2005; 2011) define algebraic reasoning as the mathematical activity characterized by the generalization of mathematical ideas, expressed in increasingly formal forms. Along the same lines, Godino et al. (2014a) point out that algebraic reasoning is directly related to processes of representation, generalization, and formalization of patterns and regularities in mathematical activity. While such reasoning evolves, progress is made in the use of different types of languages and symbolic ways to solve tasks. However, generalization from particular data is not obvious, however, this type of task is far from the classic school activity, where the formula of the general term is usually determined by stereotyped procedures (Gaita et al, 2024).

In mathematics taught in school, there are many opportunities to introduce functions and variables, and use algebraic notation from non-traditional approaches involving generalization and the search for patterns (Schliemann, Carraher & Brizuela, 2011). Teaching proportionality constitutes one such opportunity, given that it is introduced in Primary Education "by means of numerical tables and posing questions aimed at identifying the so-called homogeneous and additive properties of the proportionality function" (Burgos & Godino, 2019, p. 127). By modifying the information in the proportionality tables, suitable conditions may be generated to pose hypotheses associated with the existing relationship between the particular data shown in the table, identifying internal and external ratios, which will then allow obtaining new values to preserve the relationship and, subsequently study the general relationship given by a linear function. This way, situations associated with proportionality may contribute to the development of EAR (Burgos, 2020).

In the research of Nuñez-Gutierrez et al (2025) with high school students, results of a work carried out in a laboratory are presented, where activities focused on the concept of linear function are developed, in the search for generalization and conversion between verbal, numerical, tabular, graphical and algebraic representations. The results showed significant progress in the development of variational thinking. Consequently, it is necessary to find empirical evidence to support these assertions as early as elementary school, to use it for curriculum development and teacher training.

Therefore, we propose situations related to proportionality with information provided in tables in order to encourage students to evolve towards higher levels of EAR. To this end, we take into account didactic variables such as whether or not the data provided in the table respond to a multiplicity criterion; the number set in which the data are found; the number set to which the proportionality ratio belongs; as well as the resources used to solve the task (pencil and paper, calculator, spreadsheet). These situations are tested, and the effect of modifying the previously defined didactic variables is analyzed.

THEORETICAL FRAMEWORK

The Onto-Semiotic Approach (OSA) proposes an Elementary Algebraic Reasoning (EAR) model that considers functioning stages of mathematical knowledge applied to problem solving, where changing or modifying some variable in a task may give rise not only to new mathematical practices, but also to a progressive level of algebraization (Godino et al., 2014a).

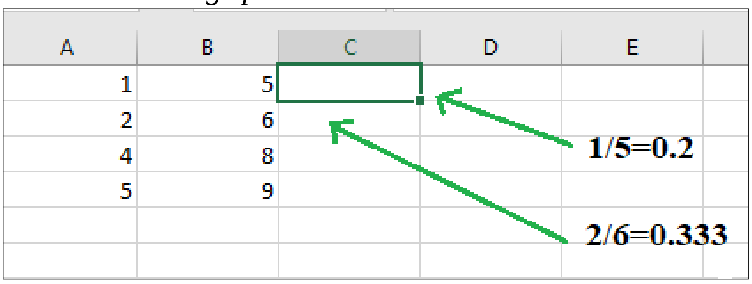
EAR levels adapted to proportionality tasks (Gaita et al..., 2023) are described as follows: *algebraization level 0* is associated with an arithmetic meaning, i.e., tasks whose solution process only considers arithmetic calculations; *proto-algebraic level 1* considers tasks requiring a process of reducing to one; *proto-algebraic level 2 is* associated with missing value tasks, i.e., tasks whose solution requires the use of unknown quantities, as well as posing and solving $Ax = B$ equations. At the *consolidated algebraization level 3,* to solve proportionality tasks modeled by tables of values, there is a need to reduce to one in the number field of positive rational numbers (Q+), i.e., either the proportionality coefficient is identified and used to determine any value, or a mathematical language is used in equivalence relations.

On the other hand, spreadsheets have been used to introduce students to algebraic task development, helping them go from specific to general thinking in terms of unknown quantities and the mathematical relations expressed in a problem (Sutherland & Rojano, 1993). Depending on how students use cells, certain characteristics may be identified associated with one of the levels of algebraic reasoning described above.

For example, if there are two columns with values and the aim is to establish their relationship by dividing two values from the same row, the following may be done:

1) Each pair of numbers is divided, and the third column is filled in with the results obtained; this is similar to the process done on a sheet of paper.

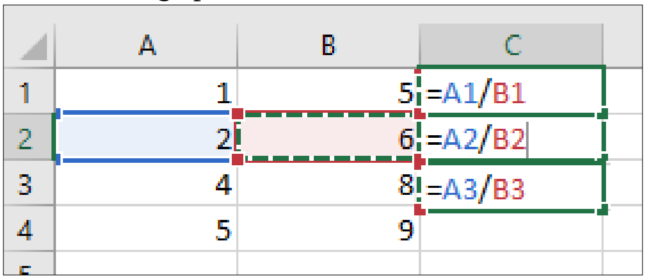
Figure 1
Using spreadsheets as a static table



2) The spreadsheet is used as a calculator that does the operation between each pair of items in the same row. The cell is selected, without copying the number in it, but the operation is repeated in each row. In other words, each operation is done independently.

Figure 2

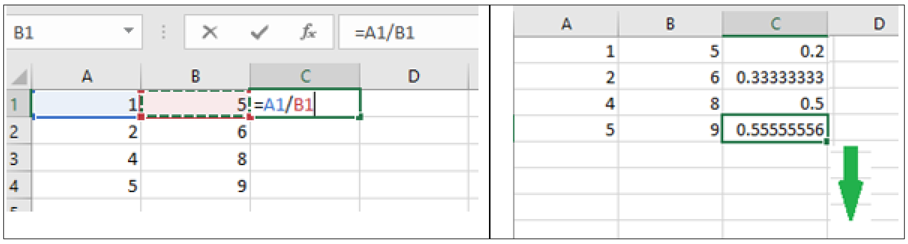
Using spreadsheets as a calculator



3) The operation is defined for the first row, referring to the cell position and not to the numerical value in it. Then, this is dragged to generate the quotients for the other rows. In other words, the operation is defined in general.

Figure 3

Using the drag-and-drop tool



In the first case, we work with specific numbers, which does not allow the identification of patterns or regularities; this type of solution is associated to a level 1 characteristic. In the second case, the result is obtained by doing operations with values from other cells, but it is defined by the name of the cell and not its specific value. When a cell is evoked by referring to its row and column, it could be considered as a quasi-variable, in the sense of Fuji & Stephens (2001), since it can serve as a bridge between arithmetic and algebraic thinking. This type of solution is associated with a level 2 characteristic, while the third case shows the manipulation of general values and operations between them to determine some missing value in a table that is directly proportional due to the context of the situation and the given values. In this case, the cell turns out to be a good approximation to the notion of variable used in algebra with symbolic notation, as proposed by Artigue (2007). Thus, the representation of the result from a cell such as a $A1/B1$ is a previous step to the use of a formal symbolic notation such as x/y, which implies not only the use of unknown quantities, but also the manipulation of these variables, an essential characteristic of level 3 reasoning.

METHODOLOGY

The research method used was didactic engineering (Godino et al., 2014b). Four phases were considered, which are described below:

- *Preliminary analysis:* A literature review was conducted, determining the theoretical elements and identifying epistemological aspects associated with the notion of proportionality.

- *A priori conception and analysis:* Didactic variables were defined, designing situations with a spreadsheet, as well as the expected behaviors characterized by an EAR evolution according to the value modification of the proposed didactic variables.

- *Implementation:* The experiment was done, observing different interactions and taking into account the dialogic-collaborative didactic model proposed by Godino & Wilhelmi (2020).

- *A posteriori analysis: results and discussion:* In this phase, different instruments were considered, such as video recordings, descriptors related to task solving, and a contrast was made between expectations and results.

A PRIORI ANALYSIS AND DESIGN

A didactic situation is designed on proportionality in a context of paint mix, modeled in tables of values where tasks previously addressed are adapted (Tourniaire & Pulos, 1985; Gaita et al., 2023). The assumption is that elementary strategies will be used to solve it, such as the following: coordinated accumulations, unit value, ratio comparison, intensive ratios, scale ratios, erroneous strategies and backwards strategies. These procedures will account for the evolution in the students' proportional reasoning (Obando et al., 2014).

Likewise, didactic variables are defined, and once modified, they will make a change in knowledge (Brousseau, 2007). Variables considered in the research refer to the characteristics of the data shown in the table and the procedures followed to solve the task. These didactic variables are the following:

- *Variable 1*: multiplicity relationship between data.

- *Variable 2*: calculation procedures, such as finding the missing term in an equality of equivalent fractions.

- *Didactic variable 3*: thoroughness and item order in the table.

- *Didactic variable 4*: relationship between values, which may be "local" between pairs of consecutive values, or up to "global", which implies determining a general formation rule.

- *Didactic variable 5*: numerical field to which the data belong, and the proportionality constant.

- *Didactic variable 6*: resource used, which may be pencil and paper or a spreadsheet.

Next, the didactic situation is presented, consisting of two tasks. In each task, a statement is presented, explaining the values taken by the didactic variables as well as the mathematical objects involved, and describing the expected mathematical behaviors.

The nature of the information presented in each task should lead students to use the spreadsheet differently in each case.

Task 1

Figure 4
Task 1 statement

Juan's mother has a hardware store where they mix paint upon customer's request. Today, a client ordered 20 liters of a blue shade which, according to Juan's mother's calculations, is the result of mixing 14 liters of blue paint and 6 liters of white paint. Juan then asks himself, "what should my mother do if another client orders different amounts of paint, but the same shade of blue? Well, Juan came up with the following table:

White (liters)		4	5	6	7	8	9	10	11	275
Blue (liters)				14						

How can you help Juan complete the table?
Explain how you completed the table.

Values taken by the didactic variables:

- *Variable 1:* Except for numbers 6 and 9, values in the table do not have a common factor.
- *Variable 2:* Specific calculations are needed to find the missing values in the cells; a spreadsheet may be used as a calculator. Recognizing the formation rule suggests reducing to one.
- *Variable 3*: The table of values is not comprehensive, as the values are presented in a scattered way and do not respond to a formation rule. It will be necessary to add values such as the unit.
- *Variable 4:* The external ratio is a positive rational value ($\frac{14}{6} = 2{,}3333\ldots$).
- *Variable 5:* Positive rational numbers (Q+).
- *Variable 6:* A spreadsheet will be used as a calculator, and variables will be inserted with cell names.

Primary objects involved:

Table 1 describes the mathematical objects involved in task 1, taking into account the OSA position on this matter.

Table 1
Primary objects and their correspondence to possible actions in task 1

Primary objects	Description
Situation	Problem of proportionality in a context of paint mixing.
Language	Natural language, numerical language, and the use of a letter to express relationships between two values.
Concepts	Definition of equivalent fractions. Definition of decimal numbers.
Proposals	The shade of the paint is kept if the quotient between the two colors used is the same.
Procedures and arguments	Division and finding the missing value for each particular case.

Expected mathematical behaviors:

Since the values in the first row are not multiples of each other, the expectation is to create the need to consider the unit in one column and a value to be determined in the second cell of that column, which can be expressed as "x", as shown in Figure 5.

Figure 5
First solution to proportionality task 1

White (liters)	**1**	4	5	6	7	8	9	10	11	275
Blue (liters)	**x**			14						

Missing value "x" may be found by using cross-multiplication as follows.

$$\frac{1}{x} = \frac{6}{14}$$
$$14 = 6x$$
$$\frac{7}{3} = x$$
$$2{,}333 \ldots = x$$

Another possible solution is to consider that the fractions are equivalent by taking $\frac{6}{14}$ as reference, using cross-multiplication without the need to get to one, and thus fill in the cells with the missing values. A spreadsheet can be used as a calculator, as shown in Figure 6.

Figure 6
Second solution to proportionality task 1

B	C	D	E	F	G	H
PintWhite paint		4	5	6	7	8
PintBlue paint		9,333	11,666	14		
4x14/6=9,333						
5x14/6 = 11,666						

The task can also be solved by using formulas involving cells and the drag-and-drop tool, as shown in Figure 7.

Figure 7
Possible solution using formulas with Excel cells

B	C	D	E	F	G
PinWhite paint		4	5	6	7
PinBlue paint		=D1*F2/F1		14	

Task 2 is presented below.

Task 2

Figure 8
Task 2 statement

Juan is thinking about ways to keep getting blue shades from different liters of white and blue paint, respectively, as shown below:

White (Liters)		4	5	6	7	8	9	10	11	275
Blue (Liters)				14						

How can you help Juan complete the table?
Explain what you did to complete the table.

Values taken by the didactic variables:

Variable 1: Values in the table are not multiples of each other.

Variable 2: Although it is possible to do specific calculations to find the missing values in the cells, it will be more efficient to do operations with the cells.

Variable 3: The table of values is not comprehensive and does not follow a formation rule. In this task, recognizing the formation rule requires, for example, reducing to one to generate the proportionality coefficient, or considering equivalent fractions to find the missing fourth term.

Variable 4: The external ratio is a positive rational value ($\frac{14}{6} = 2{,}3333\ldots$).

Variable 5: Positive rational numbers (Q+).

Variable 6: After explicitly or implicitly identifying the proportionality coefficient, a spreadsheet can be used with the drag-and-drop tool.

Primary objects involved:

Table 2 describes the mathematical objects involved in task 2, taking into account the OSA position on this matter.

Table 2
Primary objects and their correspondence to possible actions in task 2

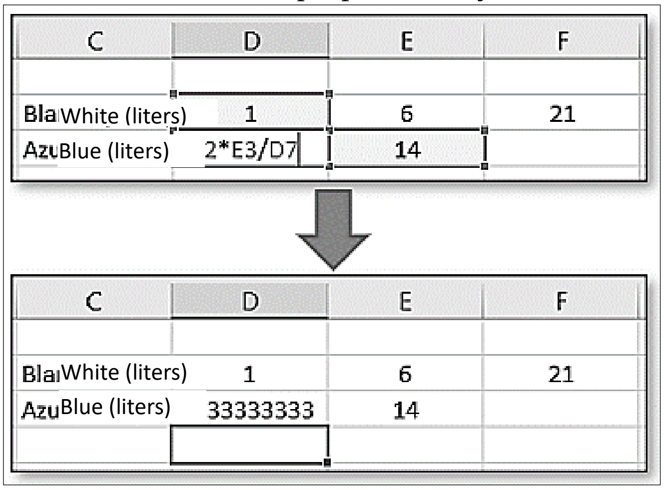
Primary objects	Description
Situation	Proportionality problem in a context of paint mixing.
Language	Natural language, numerical language, and the use of cell names to express variables.
Concepts	Definition of equivalent fractions. Definition of the proportional factor.
Proposals	The shade of the paint is kept if the quotient between the two colors used is the same.
Procedures and arguments	Obtain the proportionality coefficient. Multiply a value from the first row by the coefficient obtained and use the drag-and-drop tool to find the values in the missing cells.

Expected mathematical behaviors:

An expected solution for task 2 is to determine the proportionality factor by considering any two particular data; then, use that factor to calculate the number of liters of blue paint corresponding to one liter of white paint, i.e., reducing it to one. With that information, to obtain the values from the other columns, multiply them by the ratio 2.333333; e.g., the blank cell in column F will be filled in by multiplying

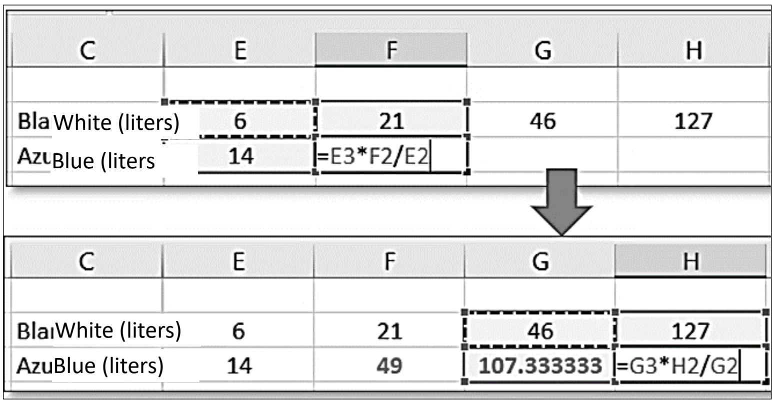
2.33333 by 21. The same strategy will be used to find the other missing values, as shown in Figure 9.

Figure 9
First solution to proportionality task 1



Another expected solution is to use the *cross-multiplication* technique to find the missing value, a procedure that is none other than the use of equivalent fractions. By using a spreadsheet, the technique is extended to $\frac{6}{14} = \frac{21}{x}$ by referring it to the cells containing the known and unknown values: $\frac{E2}{E3} = \frac{F2}{F3}$. This is represented in Figure 10.

Figure 10
Second solution to proportionality task 2



This way, the procedure is generalized since it will be valid whatever the values are in cells $E2, E3$ and $F2$.

Moreover, a means of control can be established to verify whether or not the solution is correct by dividing the values in the first row by those in the second row;

thus, obtaining equal or different quotients will help confirm if the values obtained for the missing cells are correct, as shown in Figure 11.

Figure 11

Validating answers

C	E	F	G	H	I	J
White (liters)	6	21	46	127	356	262.285714
Blue (liters)	14	49	107.333333	296.333333	830.666667	612
W/B	0.42857143	0.42857143	0.42857143	0.42857143	0.42857143	=J2/J3

To emphasize the characteristics that data in the same proportionality table must have, teachers may ask students to fill in the table with other values that are not necessarily integers.

Although autonomous validation of the solution found by filling in the tables with pencil and paper is not usually considered because it is too costly, it can be done economically by using a spreadsheet and the drag-and-drop tool in particular.

Implementation

Twenty-three students between 11 and 12 years old participated in the implementation of the didactic situation as part of their mathematics classes. Students were divided into two sessions on different days.

In the first session, several activities were carried out in relation to task 1; Table 3 briefly describes each of them.

Table 3

Didactic trajectories carried out in the development of task 1 by using an Excel spreadsheet

Session 1	
Didactic Trajectories	**Time**
• Individual reading of task 1.	5 min
• Individual work for task 1 using the spreadsheet.	40 min
• Solve a worksheet for reinforcement, including a problem similar to task 1. Students solve it in the spreadsheet and save the file.	20 min
• Closure activity done by the teacher.	5 min

In the second session, the second task was presented.

Table 4
Didactic trajectories carried out in the development of task 2 by using an Excel spreadsheet

Session 2	
Didactic Trajectories	**Time**
• Individual reading of task 2.	5 min
• Individual work for task 2 using the spreadsheet.	15 min
• Work on task 2 in groups of 4 students with the help of the spreadsheet. Group d**iscussion of the solutions obtained individually.**	35 min
• Presentation of the agreed solution by a representative of each group.	15 min
• Institutionalization of the procedures and terms associated with the proportionality relationship.	15 min
• Closure activity done by the teacher.	5 min

To collect and systematize the results, video recordings and checklists are used with descriptors related to task solving, such as whether or not the solution to the task is correct, the types of languages used, whether or not arithmetic or geometric progressions are established in the solution, the way in which the ratio is established, etc. Results are analyzed in the following section.

RESULT ANALYSIS AND DISCUSSION

In the answers to tasks 1 and 2, different solution procedures were observed, which was predictable due to the nature of the information presented in each task. This made it possible to draw some conclusions regarding the students' evolution in the EAR levels.

Results of Task 1

Table 5
Results from solving task 1

Aspects to Consider	Descriptors	f_a	%
Task solving	1. Solves task 1 correctly	21	91,30
	2. Solves task 1 incorrectly	2	8,70

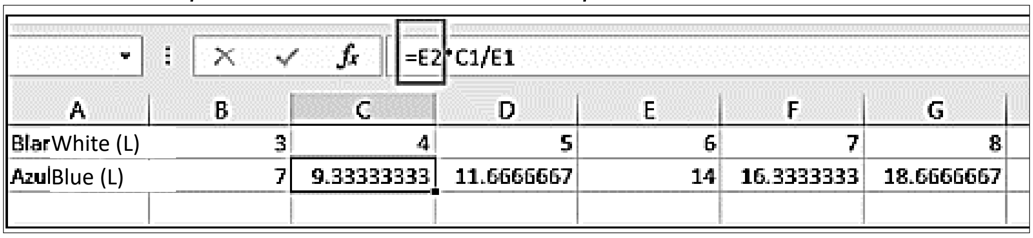
Use of language type	3.Provides information on *how* the table was constructed using Excel as a calculator by means of a numerical language	14	60,87
	4. Enters a variable using the name of the cells (e.g., B2, C3, etc.) by means of a symbolic language	14	60,87
	5. Uses symbols such as: *, =, ( ), x, / to do operations with the help of Excel	22	95,65
Relationship between values	6. Establishes the 3:7 ratio in a column or in the justification (“reducing to a minimum grouping that serves as a unit in the situation”)	15	65,22
	7. Relates the values of the two paints with “equivalent ratios” by means of cross-multiplication using Excel formulas	15	65,22
	8. Multiplies the resulting value of 14:6, which is 2.3333..., by each value of the first variable to obtain the other values of the second variable	3	13,04
	9. Relates the cell names in Excel to find the value of each cell, e.g.: =D1*F2/F1	12	52,17
	10. Establishes a relationship between numbers to find the value of each cell, e.g.: =7*4/3	11	47,83
	11. Obtains the remaining values of the second variable using the drag-and-drop tool, thus validating the constant value by obtaining the consecutive values of the quotients	1	4,35

Initial generalization processes	**12. Fill in each column whose calculations are independent of other columns**	**8**	**34,78**
	13. Establishes the process of “local” generalization by establishing the relationship between some values from the proportionality table	8	34,78
	14. Establishes a “global” generalization by filling in the proportionality table with the values from the ratio between a given fraction	14	60,87

According to the results of descriptor 4 in task 1, over 60% of the students used a symbolic language by using the names of the cells as variables. This type of procedures show characteristics of algebraic reasoning because, although cells have a particular value, when doing operations with the names of the cells, the result will be valid regardless of the value in there, as shown in Figure 12.

Figure 12

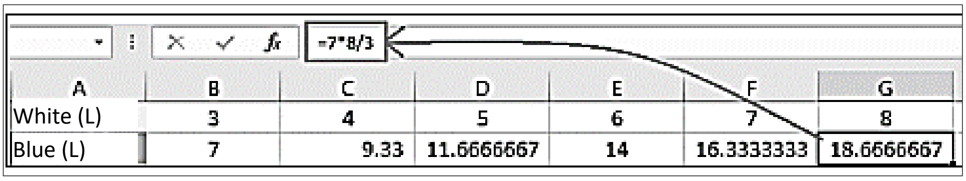
Solution of task 1 with the introduction of a variable with the cell name



The following solution also shows that the values of each cell are filled in; however, this is done from the numerical values of each cell; i.e., the spreadsheet is used as a calculator, as mentioned in descriptor 10. See Figure 13.

Figure 13

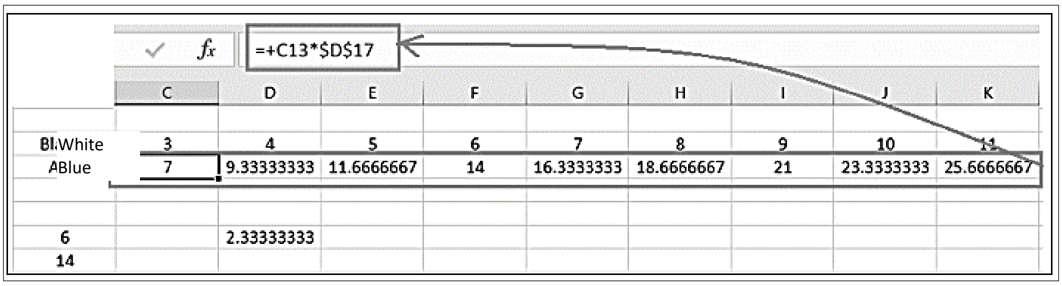
Example of descriptor 10 in task 1



Descriptor 11 in task 1 shows the solution of a student who got the missing values using the drag-and-drop tool, which corresponds to an even higher level of generalization and to characteristics of a higher level of algebraic reasoning, as shown in Figure 14.

Figure 14

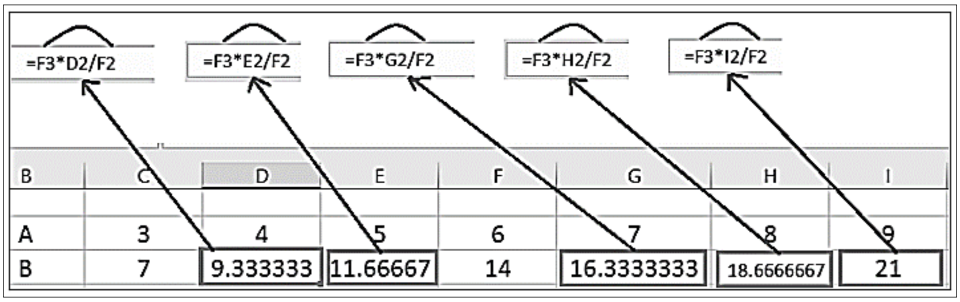
Example of descriptor 11 in task 1



Students characterized by descriptor 14 have been able to establish a global generalization since they filled out the missing values based on the ratio between a given fraction. See Figure 15.

Figure 15

Example of descriptor 14 in task 1



There is a significant difference between the mathematical practices carried out when dealing with proportionality problems with static tables (those solved with pencil and paper) and with dynamic tables (those solved by using references to the location of the cells and not to their content). Thus, in the first case, students seek to relate the missing values by doing operations with specific numbers and expect to find natural numbers as answers. As pointed out by Tourniaire & Pulos (1985), students develop proportionality tasks involving discrete quantities better than continuous ones, just as in this problem. This happens even though in previous years students did operations with decimal expressions. This implies that these expressions are not yet considered as part of the number set in which the answer to the problems is expected to be found.

Likewise, the fact that the proportionality ratio is not an integer made some students mistakenly use additive strategies or create formation rules that did not preserve proportionality, as observed by Block (2006), Block (2021) & Gaita et al. (2023).

Results of Task 2

Table 6

Results from solving task 2

Aspects to Consider	Descriptors	f_a	%
Task resolution	1. Solves task 2 correctly	33	84,62
	2. Leaves task 4 blank	0	0
	3. Solves task 2 incorrectly	2	15,38
Use of language type	4. Provides information on *how* the table was constructed by means of a natural language	30	76,92
	5. Provides information on *how* the table was constructed using Excel as a calculator by means of a symbolic language	28	71,79
	6. Uses some variable when establishing relationships between values, e.g.: B2, C5, etc.	26	66,67
Relationship between values	7. Establishes the 3:7 ratio in a column or in the justification (“reducing to a minimum grouping that serves as a unit in the situation”)	3	7,69
	8. Establishes the 1:2,333... ratio in a column or in the justification (“reducing to one”)	1	2,56
	9. Relates the values of the two paints with “equivalent ratios” by means of cross-multiplication using Excel formulas	16	41,03
	10. Multiplies the resulting value of 14:6, which is 2.3333..., by each value of the first variable to obtain the other values of the second variable	9	23,08
	11. Relates the cell names in Excel to find the value of each cell, e.g.: =D1*F2/F1	10	25,64
	12. Establishes a relationship between numbers to find the value of each cell, e.g.: =7*4/3	13	33,33

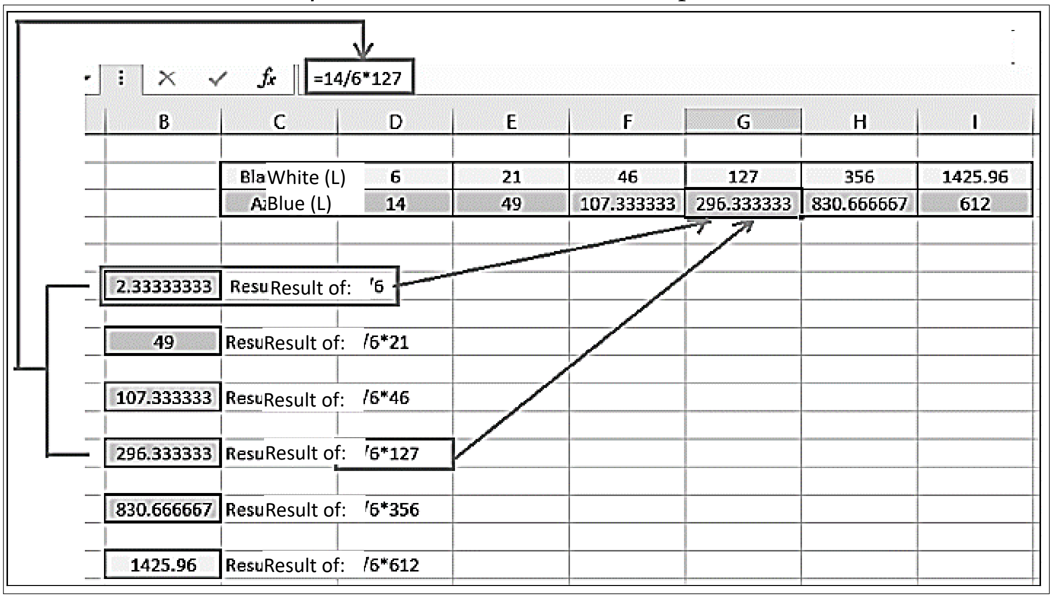
	13. Obtains the remaining values of the second variable using the drag-and-drop tool, thus validating the constant value by obtaining the consecutive values of the quotients	11	28,21
	14. Correctly fills in the empty cell that is above 612	21	53,85
Initial generalization processes	15. Fills in each column whose calculations are independent of other columns	5	12,82
	16. Establishes the process of “local” generalization by establishing the relationship between some values from the proportionality table	15	38,46
	17. Establishes a “global” generalization by filling in the proportionality table with the values	20	51,28

It can be observed that over 70% of students use more than one *language* when solving the task, using both natural and symbolic languages.

In addition, they use the spreadsheet as a calculator to do operations with specific numbers, as shown below.

Figure 16

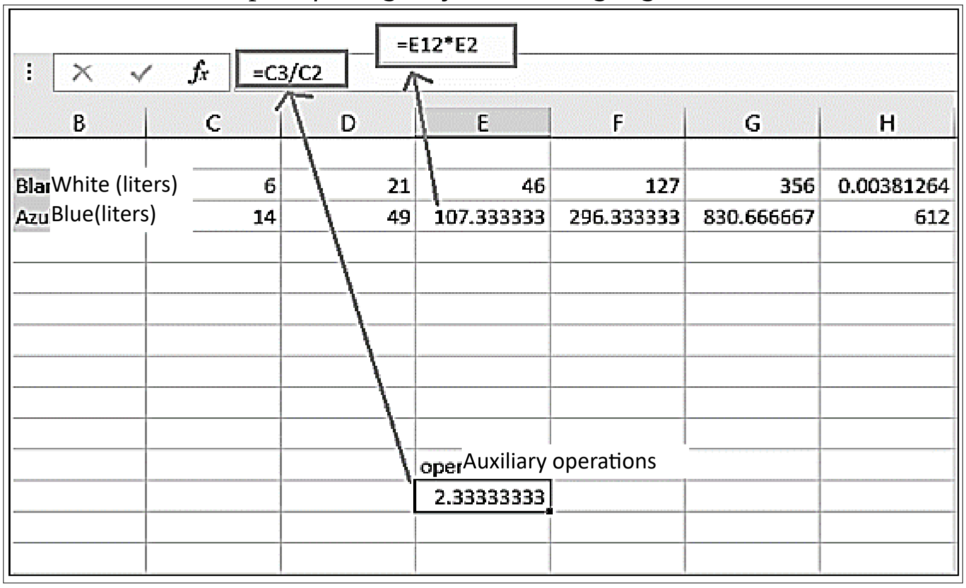
Evidence of the task 2 solution in descriptors 10 and 12



A different student uses the spreadsheet with the cells, and not with the specific value they contain, as shown in Figure 17.

Figure 17

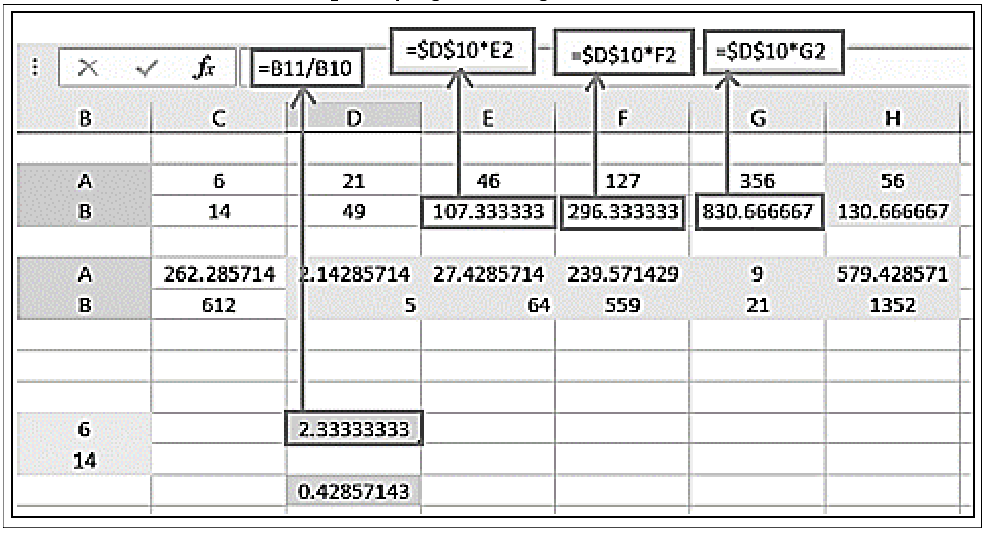
Example of using a symbolic language in task 2



Some students established a "global" generalization when filling in the proportionality table with the values based on the ratio between a given fraction. The 3:7 ratio was not used as much as in previous tasks since Excel tools allowed other strategies to be used to fill in the pivot tables. See Figure 18.

Figure 18

Example of "global" generalization



Many students found the value of each cell by relating the cell names in Excel, e.g., =D1*F2/F1. Others mistakenly used the drag-and-drop tool in the sense that the values were repeated in all cells for some students, while other students were able to fill them out with the correct values. Only student [B22] was able to use the drag-and-drop tool in such a way that it worked for any value in the table, as it set the value as, e.g., (D10), which was then used to do the calculation in the next cells.

The number field was not a problem this time. This agrees with the statement made by Gaita et al. (2023); it is possible to model proportionality by means of dynamic tables, which in this case was an Excel spreadsheet. In this regard, Araujo (2019) also argues that Excel spreadsheets allow students to recognize regularities between values in proportionality situations, as well as see data in different ways, helping them infer relationships between values and explain strategies. The latter was observed by analyzing task 2, when students were faced with this type of problem.

It is important to point out that students' answers do not entirely match the ones proposed in the *a priori* analysis. Among the coincidences, there is the use of a language and the relationship found between the values of both variables. Additionally, it was confirmed that changing the values of the didactic variables in this task allowed students to evolve their EAR.

CONCLUSIONS AND OPEN QUESTIONS

The use of spreadsheets allows students to develop mathematical practices with a higher level of algebraic reasoning. This has shown evidence of characteristics associated with different levels of EAR, which are related to the use of specific numbers, going from local to global generalizations. Using spreadsheet tools, such as freezing cells, defining operations between cells playing the role of variables, establishing the external ratio, and using the drag-and-drop tool to complete the entire table, has contributed to this.

Naming cells to do operations may be seen as a transition between the manipulation of specific values associated with numbers and the manipulation of algebraic symbols.

Spreadsheets can be used to generate activities that promote EAR development by solving tasks that involve proportionality tables, thus evolving the languages used, and the generalizations made.

By appropriately managing didactic variables, new situations can be generated demanding increasingly higher levels of algebraic reasoning, from an incipient level of algebrization (EAR 0-1) to a consolidated level of algebrization (EAR 3), which should be characteristic of high school.

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